Comparison of Mapping Operators for Unstructured Meshes

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Abstract

In this note, two PDE-based mapping operators for the generation of unstructured grids are compared. While the Winslow operator allows the construction of valid meshes for most configurations, the functional-based operators based on area, length, orthogonality and their combinations provide a finer control on the resulting meshes.

1 Introduction: Mapping Models

A widely used methodology to generate and to smooth meshes is to map an isotropic grid from parametric space onto an arbitrary domain in physical space. This can be performed by the solution of a system of partial differential equations, where the target shape in the physical domain, Ω, is imposed by the body coordinates through the boundary conditions of the PDE solved in computational space, $\mathcal{C}$.

The most frequently used operator is the Winslow set of elliptic equations ([9]) which yields smooth meshes for most conventional convex shapes. However, inappropriate mesh distributions can result for sharp concave corners where large changes of curvature occur. In [6], Knupp indicates that while the Winslow operator guarantees continuous global mapping, truncation errors can lead to folded meshes. In such instances, additional control is needed to adapt the mesh around the boundaries to insure the validity of the results, especially around high curvature parts of the physical boundary.

To increase the control over the grid quality [7] introduced a composite mapping including forcing terms to maintain orthogonality near boundaries. In [8], a new system of elliptic operators have been derived to obtain uniform cell area by controlling the Jacobian along the curvilinear coordinate directions. However, these forcing terms are difficult to choose a priori to obtain a specific mapping behavior.

A class of mapping operators that incorporates a priori information about the desired behavior can be derived using a monitor function defined to control the cells with desired properties ([1, 2, 3, 5]). These operators are based on variational smoothing methods([2]), and are derived by the minimization of appropriate grid functionals for area, length and orthogonality, and denoted as F_A, F_L and F_O, respectively. Knupp [6], Chibisov [3] and Khattri [5] have shown that a linear combination of these functionals results in valid meshes for a wide range of engineering applications.

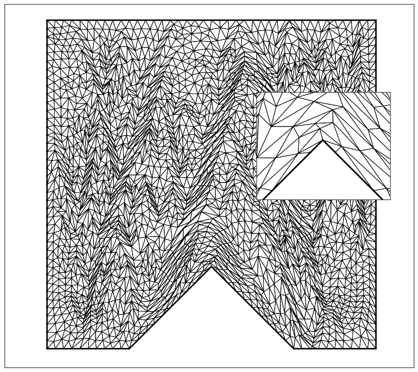
2 Numerical Discretization and Validation

Although there are clear advantages in terms of flexibility and generality in using unstructured discretizations, little attention has been addressed to extend the method to such meshes. The reason is that the Winslow operator is in non-conservative form, and therefore, the conventional integration schemes cannot be applied to unstructured meshes.

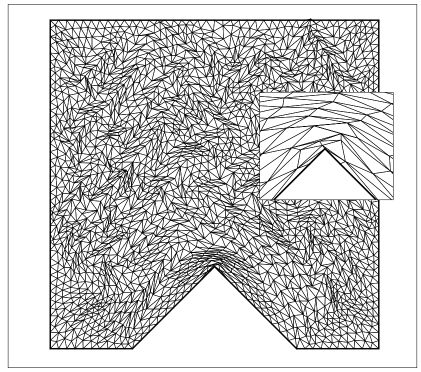
Recently, Karman [4] has introduced a finite volume discretization of the Winslow operator based on linearizing the equations. For a given point, a control volume is constructed in virtual space where the element shapes are nearly ideal.

Individual functionals and weighted combination of Length (L), Area (A) and Orthogonality (O) are studied to assess their effect on the discontinuous 45^o wedge geometry. As it can be seen from Figs. 1 (a) and (b), the area and orthogonality functionals yield non-smooth meshes, due to the non-elliptic nature of the equations, whereas length functional, Fig. 1 (c), gives smooth meshes regardless of the boundary curvature. The area-length (AL) combination shown in Fig. 1 (d) tries to overcome two important limitations of its individual functionals, which are lack of smoothness for area and degenerate cells for the length functionals. However, the solution to the AL equations fails around the tip of the wedge. A similar effect can be seen in Fig. 1 (e) for orthogonality-length (OL) due to effect of the length functional. But here more cells are folded compared to Fig. 1 (d). As it is shown in Fig. 1 (f) even though the AO meshes are not always completely satisfactory because of the non-elliptic nature of the operators, the grid smoothness is better than its individual functionals. This can be cured by a weighted combination of area and orthogonality functionals as well.

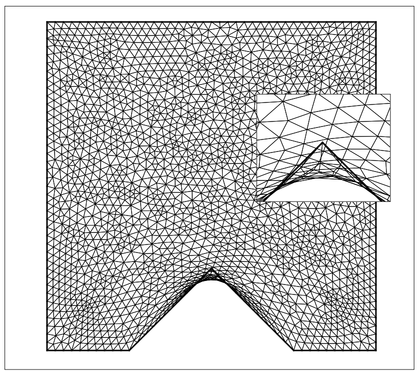
Figs. 2 illustrate the effect of different grid functionals around a four-petal configuration. As expected, the mesh obtained by the area functional (A) Fig. 2 (a) presents some discontinuities in cell sizes in the domain even though the grid is valid. The same observation can be made regarding the orthogonality functional (O) shown in Fig. 2 (b). Fig. 2 (c) confirms that the length functional (L), or Laplace equation, yields a smooth grid. However folded cells appear around the higher curvature parts of the domain. The combinations of the area-length (AL), Fig. 2 (d), and orthogonality-length (OL) functionals, Fig 2 (e), show that the effect of the length functionals in



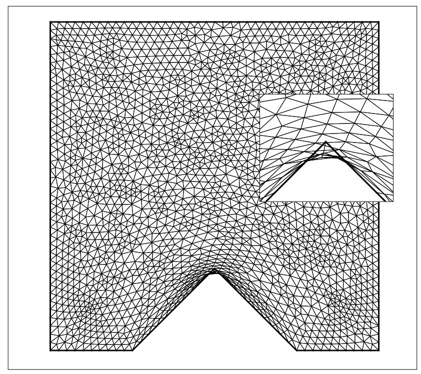
(a) Area functional



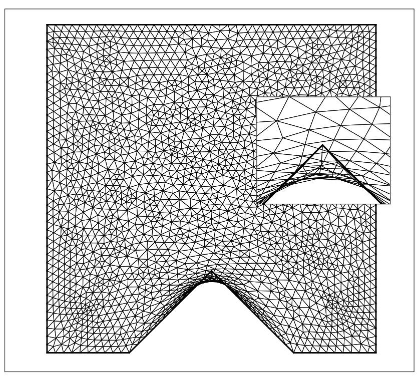
(b) Orthogonality functional



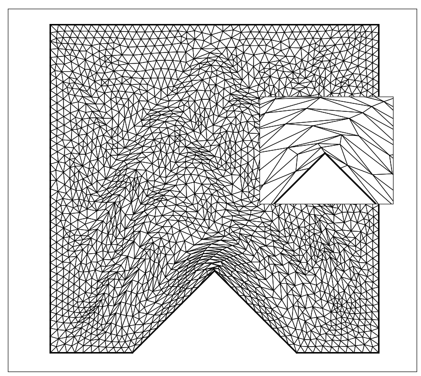
(c) Length functional



(d) Area-Length functional



(e) Length-Orthogonality functional



(f) Area-Orthogonality functional

Fig. 1. Comparison of individual and combinations of different functionals over a 45^o wedge

those equations creates inverted cells around the non-convex regions. This is because of the inherent property of the Laplace which is a pure diffusion equation.

3 Conclusion

The combination of the functional-based mapping operators and finite volume discretization scheme yields an operational method to handle complex configurations with large curvature variations, allowing the construction of valid, non-tangled, meshes. Moreover, individual and combinations of area, length and orthogonality functionals were studied and applied to two test cases. Results on both the wedge and four-petal configurations show how the final meshes are dominated by the length functional and how its absence leads to discontinuities in cell size distribution. It is suggested that non-weighted combinations of area and length (AO) can be used for a wide range of engineering applications. Usually, choosing the right values for weights ω_A, ω_L and ω_O is based on trial and error for each geometry considered. Thus using a general criteria to calculate the best combination of weights based on a distance field appears as a possible avenue to improve upon current functionals with fixed weights. This approach is currently under development.

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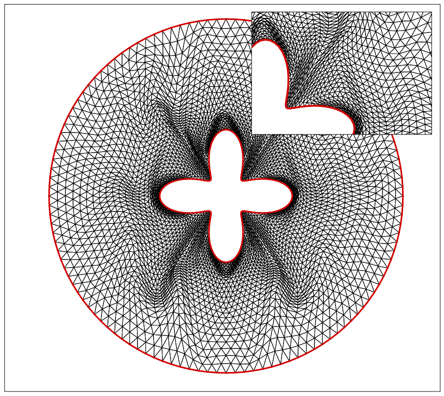
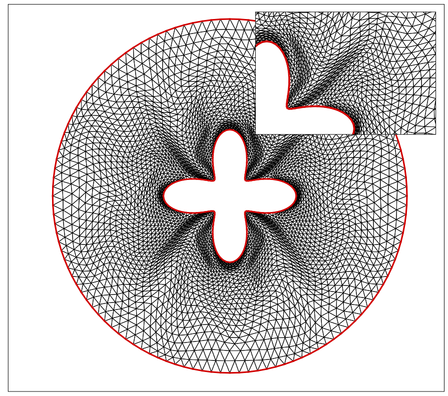
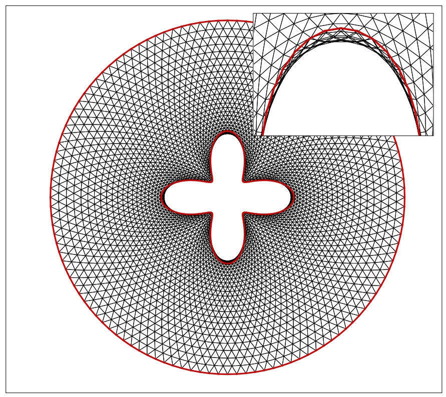


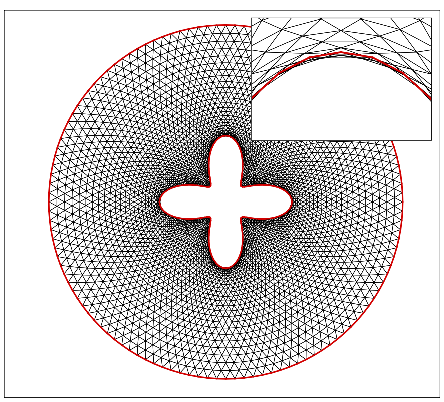
(a) 'A' , $\omega_A = 1, \omega_O = \omega_L = 0$



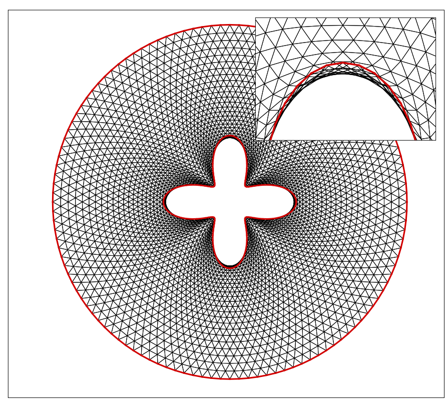
(b) 'O' , $\omega_O = 1, \omega_A = \omega_L = 0$



(c) 'L' , $\omega_L = 1, \omega_A = \omega_O = 0$



(d) 'AL', $\omega_O = 0, \omega_A = \omega_L = 0.5$



(e) 'OL', $\omega_A = 0, \omega_O = \omega_L = 0.5$

Fig. 2. Comparison of different functionals for a four-petal rose